Fractal Analysis and Lacunarity

of tractography images of the human brain.

Provata A., Katsaloulis P., Hizanidi J.

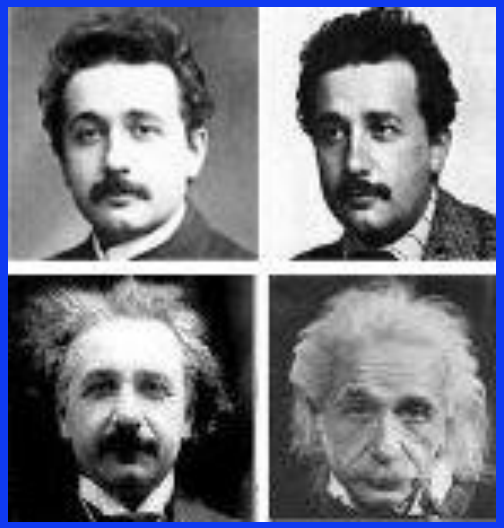
ΔΗΜΟΚΡΙΤΟΣ
ΕΘΝΙΚΟ ΚΕΝΤΡΟ ΕΡΕΥΝΑΣ ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

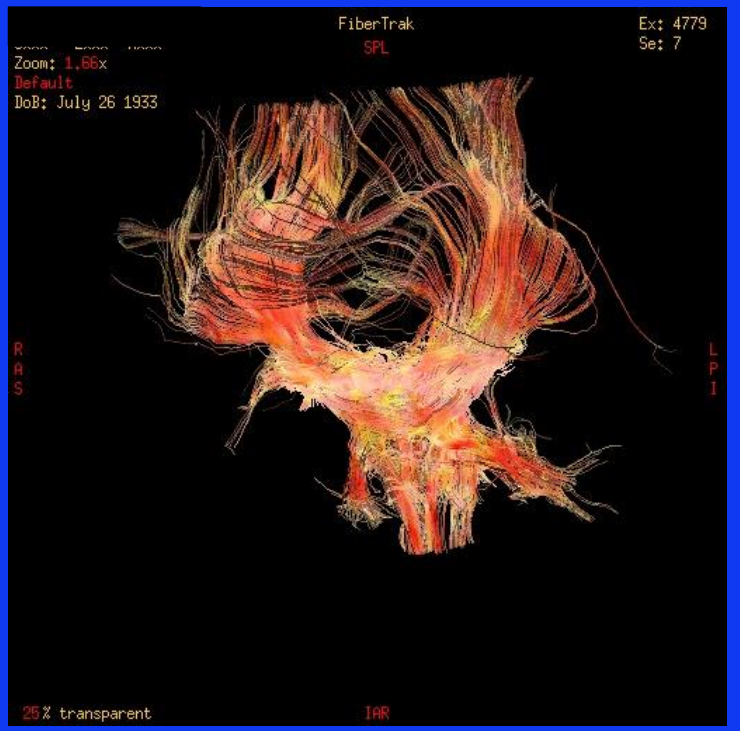
Verganelakis D.A.

EUROMEDICA

Purpose:

Study the possibility of describing neuronal tracts of human brain as fractal objects, and use the corresponding fractal dimensions as an index of pathology!

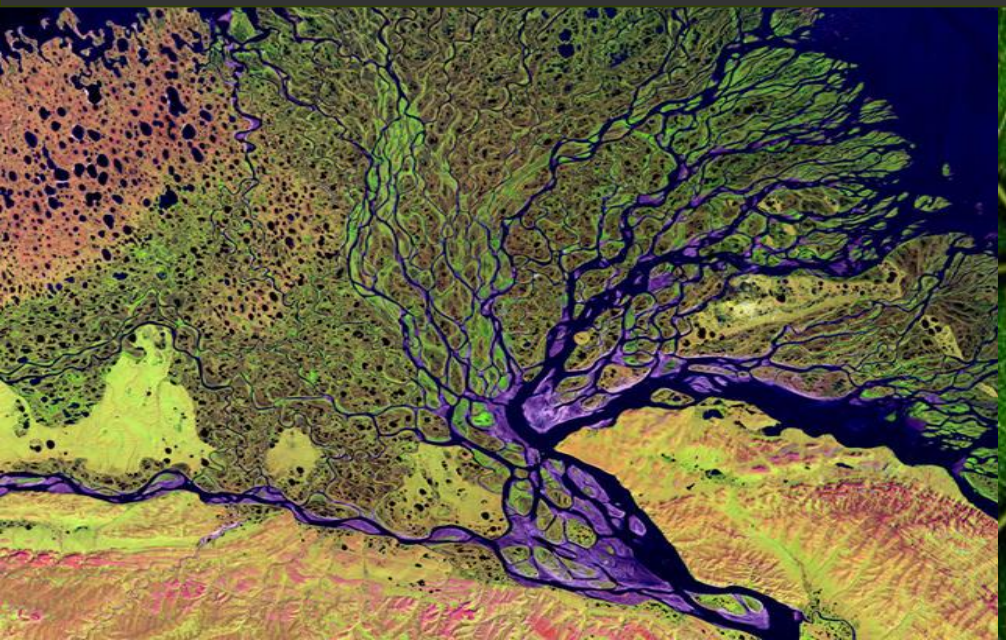


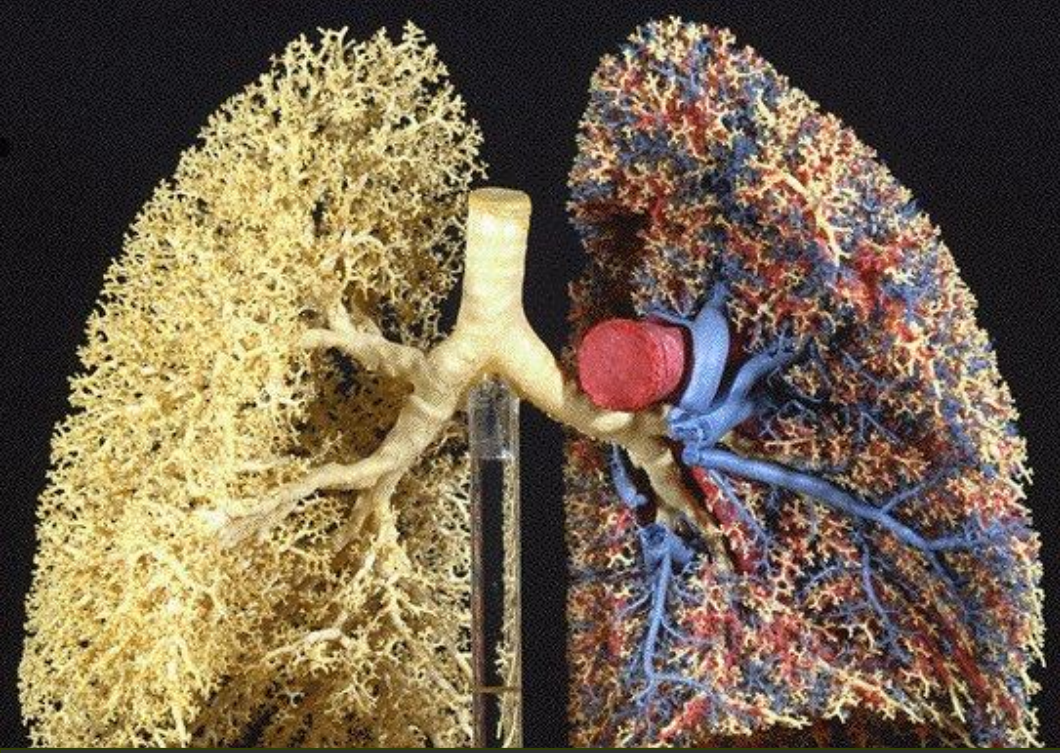


A quantitative measurement !

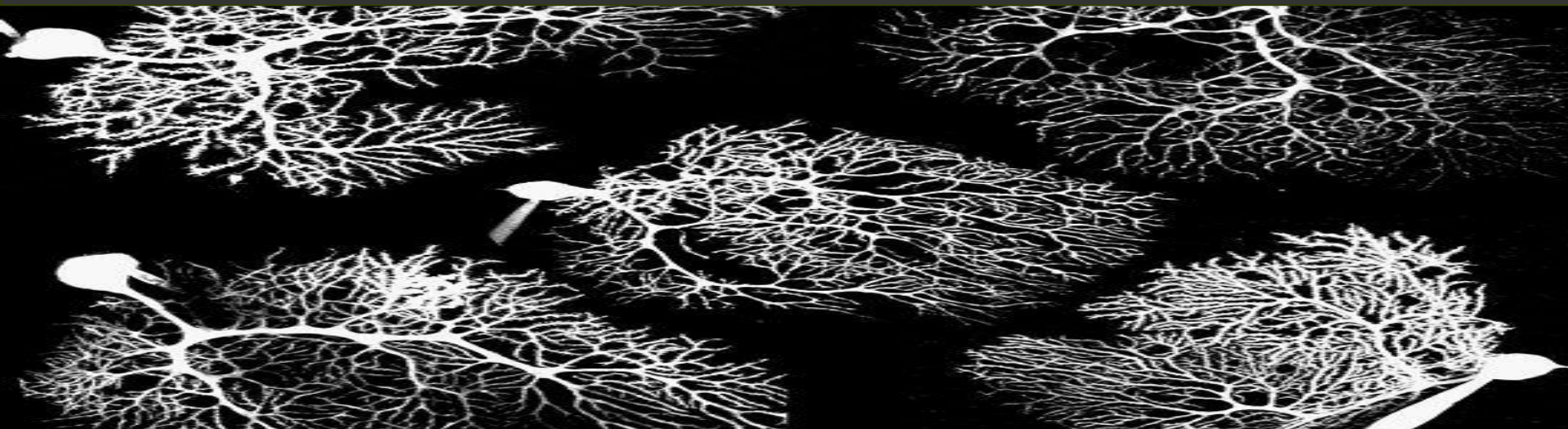


Objects with Fractal Dimensions DO exist in nature:

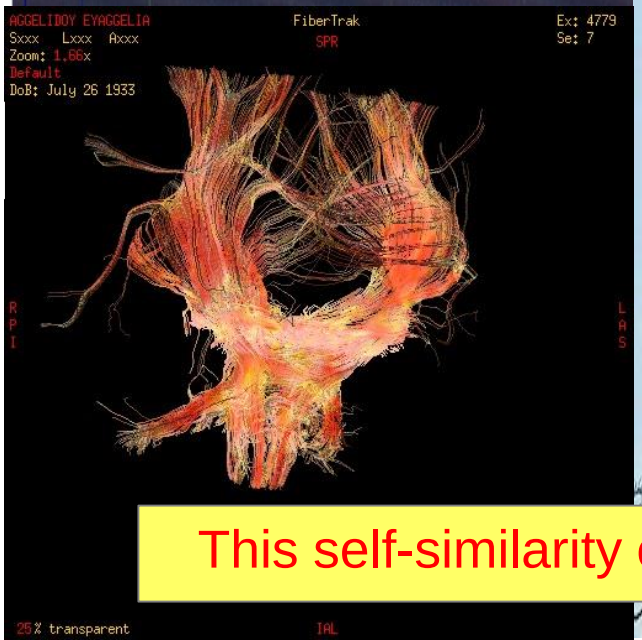


Objects with Fractal Dimensions DO exist in nature:



Fractal dimension of the brain?



Methods:

This self-similarity exists in the real world but not in all scales!!!

Dimension of an object

Common objects (mathematical and physical)

$$M = \rho V = \rho L^D$$

Fractal objects (mathematical and physical)

$$M = \rho L^{D_f}$$

with $D_f < D$

where D is the spatial dimension, M is the mass of the object and ρ is the density.

Calculation of Fractal Dimension

Hausdorff fractal dimension

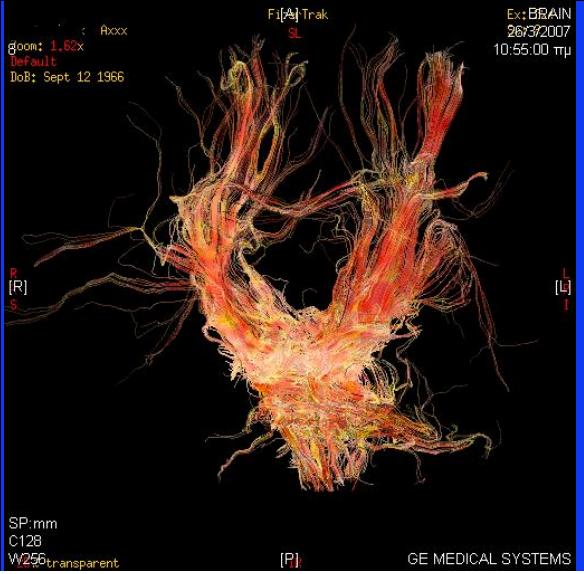
Fractal Dimension D_f is a quantitive self similarity measure

Box counting method

Divide space into equal partitions of length r

$$D_{bc} = \lim_{r \to 0} \frac{\log N(r)}{\log \frac{1}{r}} = - \lim_{r \to 0} \frac{\log N(r)}{\log r}$$

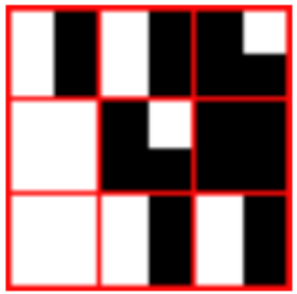
where $N(r)$ is the number of boxes of size r

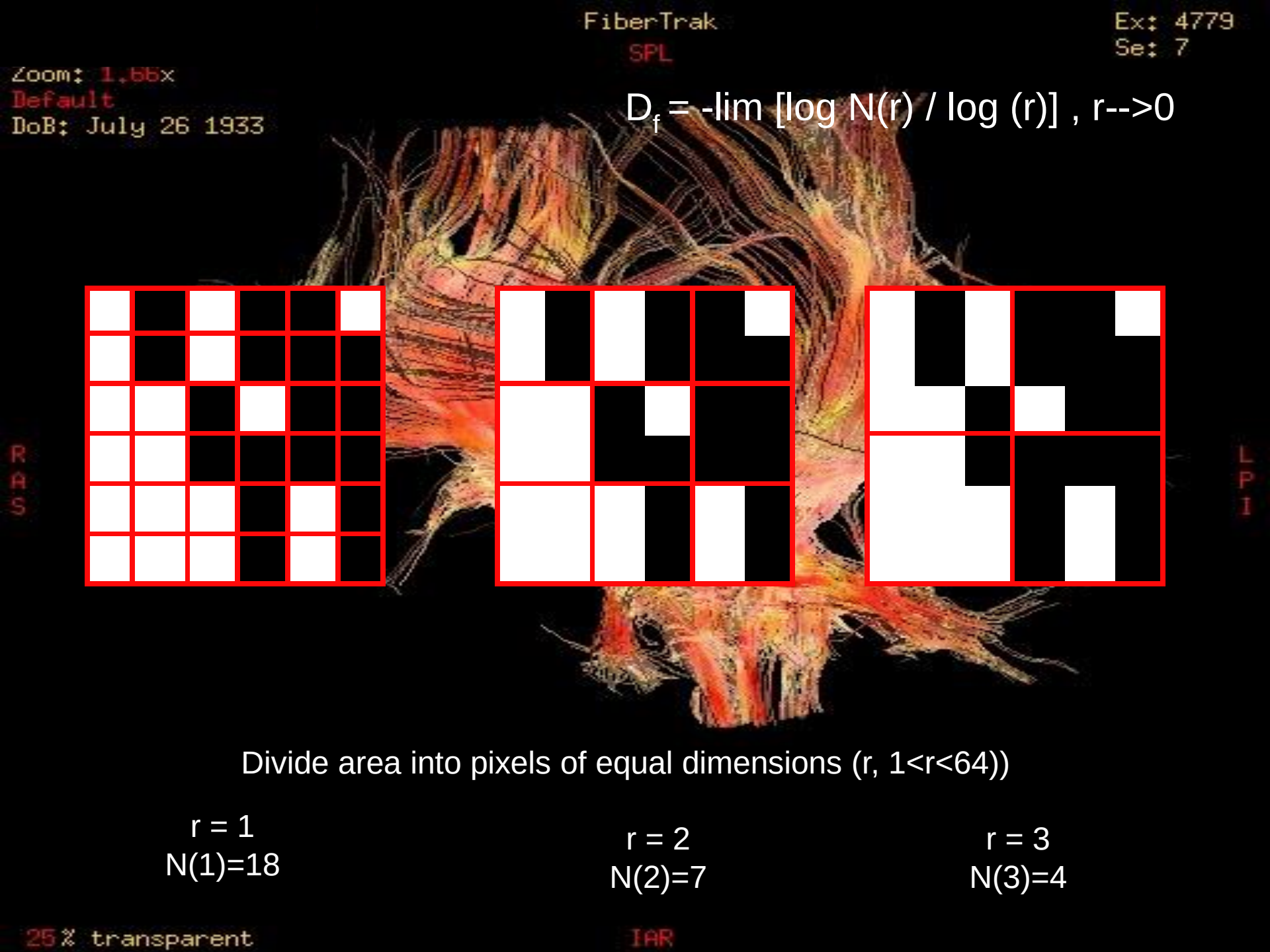


- Fractal dimension analysis requires an "existance" - "non-existance" approach.
- Images converted to inverse black & white (working grid)

Methods of estimating D_f

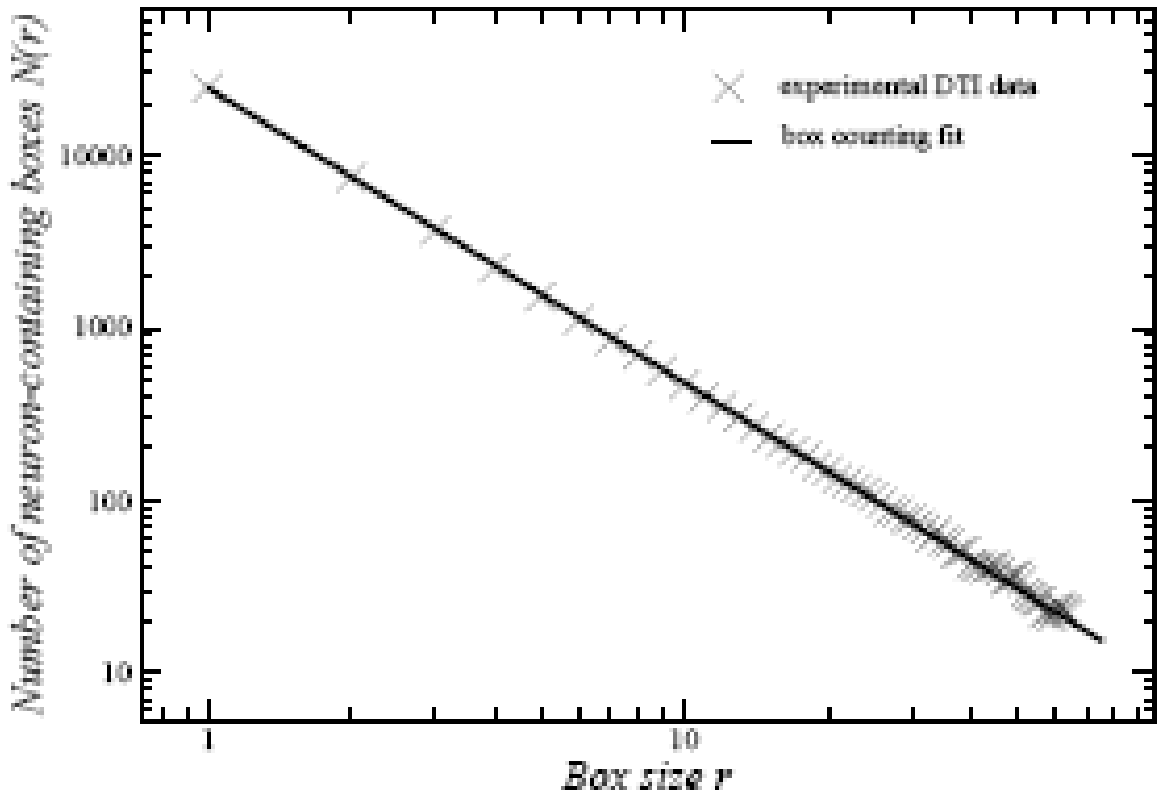
- $D_f \approx D_{bc}$: Box counting method




FiberTrak
SPL
Ex: 4779
Se: 7
Zoom: 1.66x
Default
DoB: July 26 1933
$D_f = -\lim [\log N(r) / \log (r)] , r \to 0$
R
A
S
L
P
I
Divide area into pixels of equal dimensions (r, 1<r<64))
r = 1
N(1)=18
r = 2
N(2)=7
r = 3
N(3)=4
25 % transparent
IAR

Double logarithmic: slope = D_f

Results:



$$D_f \approx D_{bc} = 1.6856$$

For purely homogeneous coverage of the 2-D space, the dimension is $D_f = 2$.

Conclusions:

2-D Fractal analysis:

Neurons from a specific ROI average value D_f: 1.58 - 1.60.

- Homogeneous between healthy subjects.
- Tracts of neurons from the same ROI imaged at different angles have the same D_f, independently of ROI and brain area.

Lacunarity analysis:

Different lacunarity values for neurons:

- originating from different parts of the brain,
- pathological vs healthy neurons.

→ HEALTHY / PATHOLOGICAL AREAS.